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Original article

Study of the influences of rotary table speed on stick-slip vibration of the drilling system

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ABSTRACT

Stick-slip vibration presents one of the major causes of drilling problems, such as premature tool failures, low drilling efficiency and poor wellbore quality. The objective of this work is to investigate the influences of rotary table speed (RTS) on stick-slip phenomenon of the drilling system. In this study, the drilling system is treated as a lumped torsional pendulum model of which the bit/rock interaction is regarded as Coulomb friction. By analyzing cases with different RTS, two types of vibrations on the bit are found: stick-slip vibration and uniform motion. With an increase in the RTS, the stick-slip vibration on the drill bit disappears once the RTS arrives at its critical value. For the cases that stick-slip vibrations occur, the phase trajectories converge toward a limit cycle. For the cases that stick-slip vibration does not appear, the drill bit tends to stabilize at a uniform motion and the phase trajectories correspond to contracting spirals observed in the phase plane.

1. Introduction

In engineering practice, vibration often causes undesirable side effects. Friction-induced vibration endangers the safety and performance of operations in a wide range of mechanical systems, such as grating brakes and chattering machine tools [1,2]. It has been estimated that about a third of the world's energy resources appear as friction in one form or another [3]. Consequently, it should be no surprise that the friction-induced vibration has attracted the attention of many researchers in the past [4–7].

One of the engineering systems that encounters friction-induced vibration is the drilling system used for oil and gas exploitation. The drilling system mainly includes the rotary table, drillstring and drill bit. The drill bit is one of the most important parts of the drilling system because it directly impacts on the rock surface. The torque driving the bit is generated at the surface by a motor and input by the rotary table. The drillstring transports the energy from the surface to the drill bit. Field measurements based on downhole or surface monitoring have indicated that drillstring exhibits severe stick-slip vibrations [8,9]. Fig. 1 shows the velocity curves of the drilling system obtained from field tests. As can be seen from the figure, severe friction-induced vibration (known as stick-slip vibration) occurs. This type of dysfunction is recognized as a major cause of drilling problems, such as premature tool failures, low drilling efficiency and poor wellbore quality [10,11].

As shown in Fig. 2, the stick-slip vibrations of drilling system are characterized by repeated alternation of stick phases where the drill bit keeps still and slip phases where the drill bit accelerates to a velocity several times of the rotary table [12]. The stick-slip phenomenon has been subjected to some early studies, including theoretical analysis and field measurement. Many theoretical models have been discussed and well documented to explain this type of dysfunction, such as model of mass-spring on a moving belt [13] and torsional pendulum model [14,15]. In order to obtain an improved understanding of the reason for

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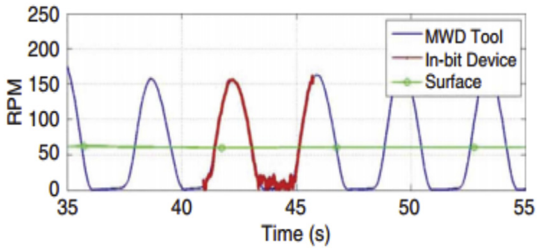

Fig. 1. Downhole measurement of rotational speed during stick-slip [9].

stick-slip vibration, laboratory drilling experiments have also been carried out [16,17]. Up to now, most of the publications have been concentrated on controlling this type of vibration by passive approaches such as bottom hole assembly optimization and use of downhole tools, or active approaches such as drilling parameter optimization based on real-time measurement and use of active control system [11,18,19]. In fact, little literature has focused on the analysis of effects of operating parameters on stick-slip vibration.

Although tremendous improvements have been achieved in fighting against the stick-slip vibration, many challenges remain. On the one hand, according to the field data, stick-slip vibration is more likely to occur in drilling deep wells due to small torsional stiffness of the drillstring [20]. On the other hand, deep wells are increasing with the development of the oil and gas industry [21]. As a result, it is of great significance to carry out investigations on the stick-slip vibration.

In this paper, the effects of rotary table speed (RTS) on stick-slip vibration of the drilling system are to be studied, which is realized by investigating the dynamic responses of the drill bit. Firstly, dynamic response of the drill bit is presented by considering the drilling system to be a lumped parameter torsional pendulum. Secondly, by referring to the actual drilling parameters, different RTS are chosen to analyze the bit dynamics. Finally, dynamic performances of the drill bit are analyzed and discussed.

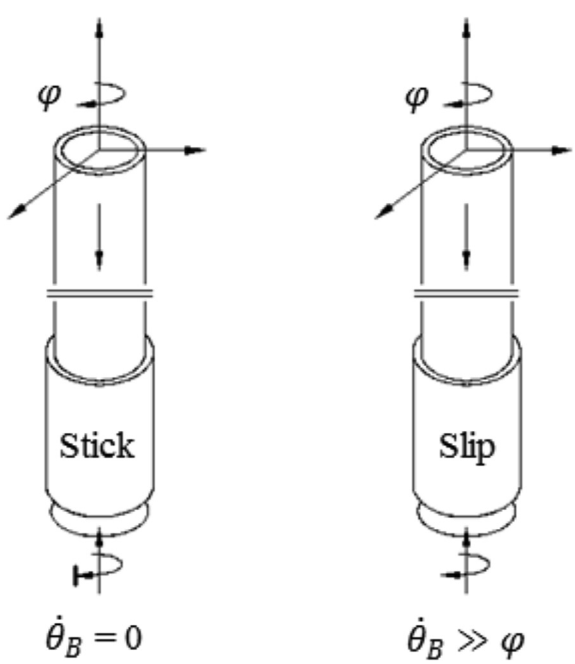

Fig. 2. Schematic diagram of stick-slip vibraion

2. Formulation of the problem

During the drilling process, energy is input by rotating a rotary table on the surface of the earth and is output via interactions between the drill bit and the formation. The drillstring that comprises of drill pipes and drill collars is used to transmit power to the drill bit for rock crushing. As a result, rotation of the drilling system is a significant factor for the drilling operation. The purpose of this paper is to find the influences of RTS on stick-slip vibration of the drilling system.

In this study, the drilling system is modeled as a lumped-parameter torsional pendulum (Fig. 3), which means the drillstring is considered as a torsional spring with its torsional stiffness K_D and moment of inertia J. The interaction between the drill bit and the formation is regarded as Coulomb friction. For the drillstring, L_P and L_C denote the lengths of the drill pipe section and drill collar section, respectively. Expressions for K_D and J are readily obtained via using knowledge of engineering mechanics [14,22].

When stick-slip phenomenon occurs on the bit, stick and slip motions alternate. In this study, for the convenience of analysis, the drill bit is assumed to be in a critical state where the stick phase ends and the slip phase is incited. That is to say, the driven torque transmitted by the drillstring overcomes the frictional torque acted on the bit and thus the bit starts to rotate at the time $t = 0$. According to the field drilling applications, we assume that the rotary table rotates clockwise with a constant angular velocity φ. Consequently, the drill bit rotates clockwise at the initial stage of the slip phase.

For the condition stick-slip vibration occurs, energy transitions between kinetic energy and potential energy in the drillstring are common. In order to obtain the equation of motion of the drilling system, both the damping force and frictional force are treated as external forces. As a result, the Euler–Lagrange equation can be used to express the motion of the drilling system:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_B}\right) = \frac{\partial T}{\partial \theta_B} + Q \tag{1}$$

where $T = \frac{1}{2}J\dot{\theta}_B^2$ is the kinetic energy of the drilling system; θ_B is the angular displacement of drill bit; the dot denotes the

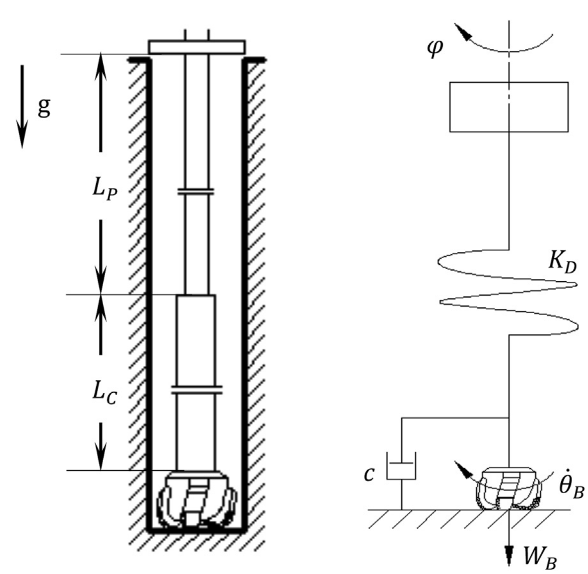

Fig. 3. Analytical model of the drilling system [22].

derivative with respect to time t; Q is the external force. And then, the bit motion is described by the following equation:

$$J\ddot{\theta}_B + c\dot{\theta}_B + K_D(\theta_B - \varphi t) + \mu_K W_B \overline{R}_B = 0 \tag{2}$$

where c is the damping coefficient; W_B is the weight on bit; μ_K is the kinetic friction which is assumed to be less than the static friction coefficient μ_S, that is $\mu_S - \mu_K > 0$; and $\overline{R}_B$ denotes the equivalent radius of drill bit obtained by using the integral method, which can be expressed as

$$\overline{R}_B = \int_0^{R_B} \frac{2\pi r}{\pi R_B^2} \cdot r \cdot dr = \frac{2}{3} R_B \tag{3}$$

where R_B is the actual radius of drill bit. It should be noted that this expression is different from these listed in most of the existing literature [23–26].

The initial condition for the drill bit moves in the slip phase can be expressed as

$$\theta_{B0} = \mu_S W_B \overline{R}_B / K_D \tag{4}$$

$$\dot{\theta}_{B0} = 0 \tag{5}$$

Since the rotary table keeps on rotating, only the relative motion between the rotary table and the drill bit, $\theta_{Br} = \theta_B - \varphi t$ and $\dot{\theta}_{Br} = \dot{\theta}_B - \varphi$, are to be analyzed in this study. The relative angular displacement response for the bit can be determined by the equation

$$\theta_{Br} = A e^{-\xi \omega_n t} sin\left(\sqrt{1-\xi^2}\,\omega_n t + \psi\right) - \mu_K W_B \overline{R}_B / K_D \tag{6}$$

where $\omega_n = \sqrt{K_D/J}$ is the natural frequency; $\xi = c/2J\omega_n$ is the damping ratio; and A and ψ are parameters determined by θ_{B0} and $\dot{\theta}_{B0}$, which can be expressed as

$$A = \sqrt{\left(\theta_{B0} + \frac{\mu_K W_B \overline{R}_B}{K_D}\right)^2 + \left[\frac{\varphi - \dot{\theta}_{B0} + \xi\omega_n\left(\theta_{B0} + \frac{\mu_K W_B \overline{R}_B}{K_D}\right)}{\sqrt{1-\xi^2}\,\omega_n}\right]^2} \tag{7}$$

$$\psi = \arctan \frac{\sqrt{1-\xi^2}\,\omega_n\left(\theta_{B0} + \frac{\mu_K W_B \overline{R}_B}{K_D}\right)}{\varphi - \dot{\theta}_{B0} + \xi\omega_n\left(\theta_{B0} + \frac{\mu_K W_B \overline{R}_B}{K_D}\right)} \tag{8}$$

3. Dynamics of a drilling system with different RTS

As can be seen from Eqs. (6)–(8), the bit dynamics are related to the RTS. Then, the relationship between the RTS and the stick-slip vibration is what we are interested. In order to reveal the influences of RTS on stick-slip vibration of the drilling system, different RTS are analyzed based on a drilling system of which the system parameters are determined by referring to the actual drilling conditions. For the drilling system with the drillstring length L_D = 3000 m, the system parameters are presented in Table 1.

Table 1
Parameters of the drilling system.

Parameter	Value	Parameter	Value
L_D (m)	3000	R_B (mm)	108
L_P (m)	2800	W_B (kN)	160
L_C (m)	200	ρ (kg/m^3)	7850
D_P (mm)	127	G (Pa)	8.0×10^{10}
d_P (mm)	108.6	μ_S	0.8
D_C (mm)	165.1	μ_K	0.5
d_C (mm)	57.2	ξ	0.1

In this paper, the RTS φ = 80 rpm, φ = 100 rpm, φ = 120 rpm, φ = 140 rpm and φ = 160 rpm are analyzed. For these cases, the corresponding bit dynamic responses can be obtained.

Case 1: φ = 80 rpm

$$\theta_{Br} = -12e^{-0.164t} sin(1.63t + 1.03) - 17.16 \tag{9}$$

Case 2: φ = 100 rpm

$$\theta_{Br} = -12.73e^{-0.164t} sin(1.63t + 0.94) - 17.16 \tag{10}$$

Case 3: φ = 120 rpm

$$\theta_{Br} = -13.50e^{-0.164t} sin(1.63t + 0.87) - 17.16 \tag{11}$$

Case 4: φ = 140 rpm

$$\theta_{Br} = -14.33e^{-0.164t} sin(1.63t + 0.80) - 17.16 \tag{12}$$

Case 5: φ = 160 rpm

$$\theta_{Br} = -15.29e^{-0.164t} sin(1.63t + 0.74) - 17.16 \tag{13}$$

Eqs. (9)–(13) show the bit dynamic responses of the drill bit for different RTS. After analyzing these responses, we found that the drill bit does not rotate anticlockwise in the slip phase, which means Eq. (2) is reasonable to describe the bit motion in the slip phase. In addition, it is revealed that two types of motion, stick-slip and uniform rotation, occur on the bit. For different RTS cases, the periodic characteristics are listed in Table 2, where T_S indicates the period of the stick phase, T_K indicates the period of the slip phase and $T = T_S + T_K$ indicates the period of the stick-slip vibration.

As presented in Table 2, stick-slip vibration occurs for only three of the five RTS. For the cases φ = 80 rpm, φ = 100 rpm and φ = 120 rpm, stick-slip vibration of the drilling system occurs. For the cases φ = 140 rpm and φ = 160 rpm, stick-slip vibration does not show. For the cases that stick-slip vibration occurs, with an

Table 2
Period characteristics of the drill bit motion.

φ (rpm)	80	100	120	140	160
T_S (s)	2.63	2.81	3.03	/	/
T_K (s)	2.0	1.51	1.12	/	/
T (s)	4.63	4.32	4.15	/	/

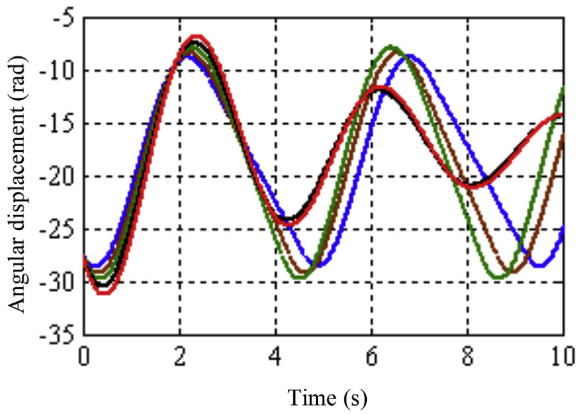

Fig. 4. Angular displacement of the bit relative to the rotate table: blue, $\varphi = 80$ rpm; brown, $\varphi = 100$ rpm; green, $\varphi = 120$ rpm; black, $\varphi = 140$ rpm; red, $\varphi = 160$ rpm.

increase in the RTS, T_S increases but T_K decreases, leading to a decrease in the period of stick-slip vibration T.

4. Results and discussion

Fig. 4 shows the time history of relative angular displacement θ_{Br}. Because the drill bit rotates behind the rotary table, and thus θ_{Br} is going to be negative. Fig. 5 shows the time history of relative angular velocity $\dot{\theta}_{Br}$. As can be seen from Fig. 4, the displacement that the drill bit lags behind the rotary table (absolute value of θ_{Br}) increase at the initial stage. This is because the rotary table keeps rotating at a constant speed and the drill bit speeds up to rotate clockwise with its initial velocity $\dot{\theta}_{B0} = 0$, so $|\theta_{Br}|$ increases before the drill bit speed reaches the RTS. After a period of time, $|\theta_{Br}|$ will reduces once the drill bit rotates faster than the rotary table. During the process, the drill bit is in a state of accelerated motion. Reduce in the $|\theta_{Br}|$ leads to a decrease in the driven torque and increase in the $\dot{\theta}_{Br}$ leads to an increase in the damping force, and therefore the drill bit steps into a state of decelerated motion once the drill bit reaches its maximum velocity.

During the slip phase, the forces act on the drill bit include: friction force, damping force and driven force, where the driven force and damping force are proportional to the $|\theta_{Br}|$ and $\dot{\theta}_{Br}$, respectively. After the accelerated rotation and decelerated rotation of the drill bit, the drill bit velocity will reduce to a value equals to 0. In the circumstances, the driven torque corresponds to $\dot{\theta}_{Br} = 0$ is smaller than the maximum static frictional torque, leading to a stationary state of the drill bit. Which also means that the drill bit does not rotate anticlockwise for the cases analyzed in this paper and that the sign of frictional torque $\mu_K W_B \overline{R}_B$ in Eq. (2) is reasonable. During the stick phase, the displacement that the drill bit lags behind the rotary table $|\theta_{Br}|$ increases due to the constant rotation of the rotary table.

As shown in Figs. 4 and 5, stick-slip vibration occurs for the cases $\varphi = 80$ rpm, $\varphi = 100$ rpm and $\varphi = 120$ rpm only. For these cases, amplitudes of the θ_{Br} and $\dot{\theta}_{Br}$ increases slightly with an increase in the RTS. As can be seen from Fig. 5, amplitudes of the drill bit velocity arrive at a value 1.6–2.0 times bigger than that of the rotary table, which is in agreement with the drilling data [27,28]. For the cases $\varphi = 140$ rpm and $\varphi = 160$ rpm, the drill bit is in a damped motion and tends to stabilize at a uniform motion. As a result, the risk of stick-slip vibration decreases with increasing the RTS, which confirms what is claimed by Refs. [29,30].

The time history of angular acceleration of the drill bit is presented in Fig. 6. As can be seen from the figure, stick-slip vibration occurs for the cases $\varphi = 80$ rpm, $\varphi = 100$ rpm and $\varphi = 120$ rpm only. For the slip phase of these cases, the angular accelerations of the drill bit are analogous to simple harmonic motion. For the stick phase, the drill bit stops rotating. However, both the relative angular displacement and driven torque increase. Since no rotation occurs, the angular accelerations of the drill bit in the stick phase are characterized by dotted lines. For the cases $\varphi = 140$ rpm and $\varphi = 160$ rpm, the angular velocity vibrates damply and then tends to stay at a value equals to zero, which means the drill bit will keep still after the initial damped motion.

Fig. 7 shows the time history of the driven torque acts on the drill bit. It is worth noting that the driven torque is the product of the relative displacement $|\theta_{Br}|$ and the torsional stiffness of drillstring K_D. For a given drillstring (including the structure and length), K_D is a fixed value. As the relative displacement θ_{Br} is negative, plot of the driven torque presents an inverse shape compares to the Fig. 4 For the cases which stick-slip occurs, the driven torque uniformly increases to a value equals to the maximum static frictional torque in the stick phase, which also means the driven torque and the frictional torque are equal in magnitude but opposite in direction. For the cases $\varphi = 140$ rpm

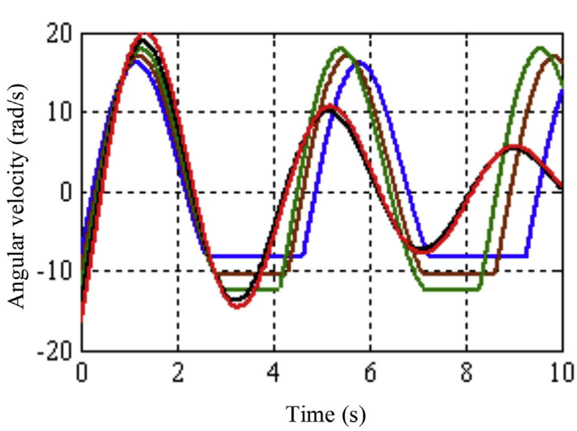

Fig. 5. Angular velocity of the bit relative to the rotate table: blue, $\varphi = 80$ rpm; brown, $\varphi = 100$ rpm; green, $\varphi = 120$ rpm; black, $\varphi = 140$ rpm; red, $\varphi = 160$ rpm.

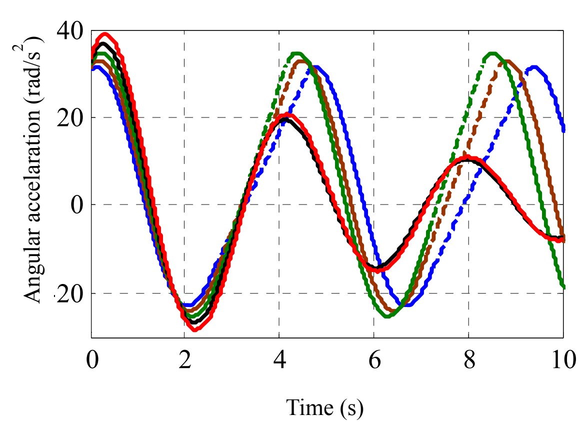

Fig. 6. Angular acceleration of the drill bit: blue, $\varphi = 80$ rpm; brown, $\varphi = 100$ rpm; green, $\varphi = 120$ rpm; black, $\varphi = 140$ rpm; red, $\varphi = 160$ rpm.

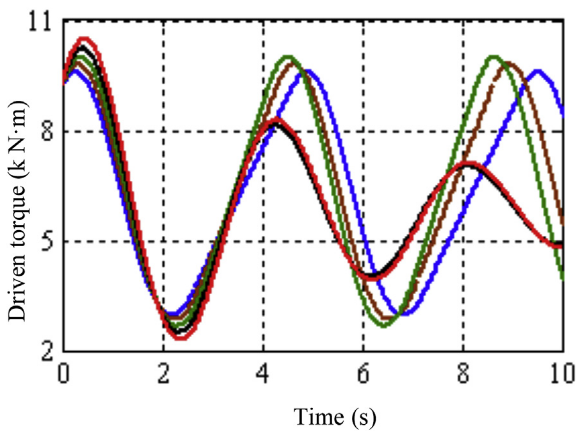

Fig. 7. Driven torque on the drill bit: blue, $\varphi = 80$ rpm; brown, $\varphi = 100$ rpm; green, $\varphi = 120$ rpm; black, $\varphi = 140$ rpm; red, $\varphi = 160$ rpm.

and $\varphi = 160$ rpm, the driven torque vibrates damply and then tends to stay at a value equals to the kinetic frictional torque.

Fig. 8 presents the time history of frictional torque on the bit. From the figure, it is easy to find the period characteristics of the drill bit responses for the cases stick-slip vibration occurs. For the cases $\varphi = 80$ rpm, $\varphi = 100$ rpm and $\varphi = 120$ rpm, the frictional torques on the bit are discontinuous, or rather segmented. As the drill bit is in the slip phase, the frictional torque on the bit is equal to $\mu_K W_B \overline{R}_B$, corresponding to a horizontal line in the figure. Stick phase starts once the slip phase finishes, corresponding a frictional torque on the drill bit shifts from kinetic frictional torque to static frictional torque whose magnitude is determined by the relative angular displacement of the critical state. During the stick phase, frictional torque equals to the driven torque as the drill bit is in a state of force balance. The stick phase ends once the frictional torque reaches the maximum static frictional torque, then the frictional torque drops abruptly to the kinetic frictional torque. For the cases $\varphi = 140$ rpm and $\varphi = 160$ rpm, the frictional torque maintains the magnitude of kinetic frictional torque.

Fig. 9 shows the phase plane of dynamic responses of the drill bit under different conditions. As can be seem from the figure, two types of motion are observed: stick-slip vibration and damped vibration. For the cases $\varphi = 80$ rpm, $\varphi = 100$ rpm and $\varphi = 120$ rpm, stick-slip vibrations take place and the phase trajectories converge toward limit cycles. For each limit cycle, it

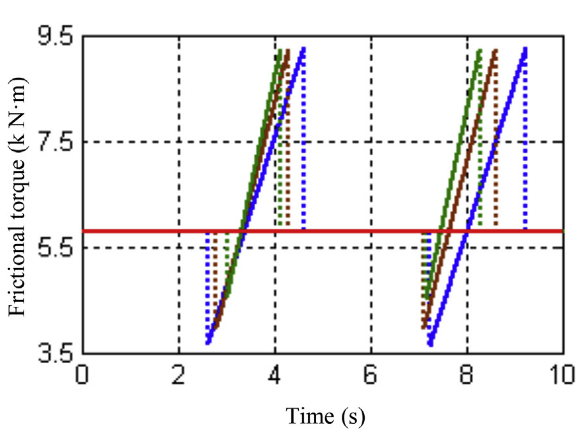

Fig. 8. Frictional torque on the drill bit: blue, $\varphi = 80$ rpm; brown, $\varphi = 100$ rpm; green, $\varphi = 120$ rpm; black, $\varphi = 140$ rpm; red, $\varphi = 160$ rpm.

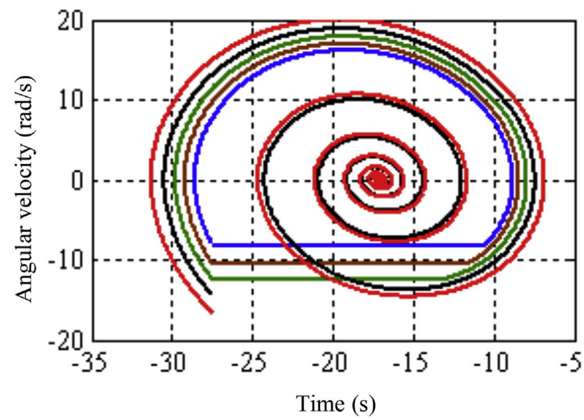

Fig. 9. Phase trajectory of the relative motion of the bit: blue, $\varphi = 80$ rpm; brown, $\varphi = 100$ rpm; green, $\varphi = 120$ rpm; black, $\varphi = 140$ rpm; red, $\varphi = 160$ rpm.

consists of a curve represents the slip phase and a horizontal line represents the stick phase. However, an unexpected and important feature of the system is that the amplitude of the stable limit cycle increases with increasing RTS, which means the stick-slip vibration aggravates. For some higher constant RTS, drill bit motion without stick-slip appears. For the cases $\varphi = 140$ rpm and $\varphi = 160$ rpm, the drill bit vibrates damply at the initial stage and tends to stabilize at uniform motions (constant velocity at both the rotary table and the drill bit), corresponding to contracting spirals observed in the phase plane.

Although the above results are specific to a particular drilling system with parameters listed in Table 1, the analytical method remains general. Its validity with respect to the actual drilling application is contingent upon the adequacy of the analytical model and also the input parameters. Then, what we are interested in is the critical RTS. For the drilling system studied in this paper, the critical RTS is found to be 127 rpm, which means stick-slip vibration occurs on the condition that $\varphi < 127$ rpm and stick-slip vibration disappears by increasing the RTS to a value $\varphi > 127$ rpm. What should be stressed is that the critical value will change for different drilling system or drilling parameters and that the critical values can be obtained by referring to the method in this paper. In fact, increasing the RTS is a common approach to avoid stick-slip vibration in the actual drilling operations. By using the analysis in this paper, the mechanism of this approach can be easily interpreted.

5. Conclusions

(1) Based on a lumped-parameter torsional pendulum model of the drilling system, equation of motion of the drill bit is established and solution of the equation is obtained. In this paper, five dynamic responses of the drill bit are analyzed by referring to the actual drilling conditions.
(2) For the five RTS, two types of motion are observed: stick-slip vibration and damped vibration. For the cases $\varphi = 80$ rpm, $\varphi = 100$ rpm and $\varphi = 120$ rpm, stick-slip vibrations take place. With an increase in the RTS, T_S increases but T_K decreases, leading to a decrease in the period of stick-slip vibration T.
(3) For the cases that stick-slip vibration occurs, both the angular displacement and angular velocity increase slightly with increasing the RTS. The amplitude of the angular velocity of the drill bit is 1.6–2.0 times bigger than the RTS, which agrees with the drilling field.

(4) The driven torque on the bit is determined by the θ_{Br} for a given drilling system and the frictional torque relates to the state of motion of the drill bit. For the cases $\varphi = 80$ rpm, $\varphi = 100$ rpm and $\varphi = 120$ rpm, the frictional torques on the bit are segmented. Both the driven torque and the frictional torque fluctuate slightly with an increase in the RTS.
(5) Two types of motion are observed for the five cases: stick-slip vibration and damped vibration. For the cases $\varphi = 80$ rpm, $\varphi = 100$ rpm and $\varphi = 120$ rpm, the phase trajectories converge toward limit cycles. For the cases $\varphi = 140$ rpm and $\varphi = 160$ rpm, the drill bit tends to stabilize at uniform motions (constant velocity at both the rotary table and the drill bit), corresponding to contracting spiral observed in the phase plane.
(6) For the drilling system studied in this paper, the threshold value of RTS is found to be 127 rpm, which means stick-slip vibration occurs on the condition that $\varphi < 127$ rpm.
(7) Occurrence of stick-slip vibration is not necessary, which depends on the drilling system chosen for the analysis. When looked into more details, many factors play important role in this motion, such as rock formation type, drill bit type, weight on bit, drillstring length, and damping. Consequently, it is possible to mitigate the stick-slip phenomenon by properly matching these parameters.

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